Math Skills Assessment

1 Functions, Graphs, and Equations

1. For each of the following functions, plot enough points to sketch a complete graph. Describe where the function is increasing and where it is decreasing. Find the local maxima and minima:

   (a) $y = 3x - 2$;
   (b) $y = x^2 + 1$;
   (c) $y = x^3 - x$;
   (d) $y = |x|$;

2. Find the equation for the linear function whose graph:

   (a) has slope 2 and y-intercept (0, 3),
   (b) has slope -3 and y-intercept (0, 0),
   (c) has slope 4 and goes through the point (1, 1),
   (d) goes through the points (2, 3) and (4, 5),
   (e) goes through the points (2, -4), and (0, 3).

3. Suppose $F(q)$ is the revenue generated from producing q units of output. Give an economic interpretation for the *slope* of $F(q)$.

4. Solve the following systems of equations:

   (a)

   $$\begin{aligned} 3x + 3y &= 4 \\ x - y &= 10 \end{aligned}$$

   (b)

   $$\begin{aligned} 4x + 2y - 3z &= 1 \\ 6x + 3y - 5z &= 0 \\ x + y + 2z &= 9. \end{aligned}$$

5. Consider a matrix with n rows and m columns, with each element of the matrix being denoted by a_{ij} where i corresponds to row i and j corresponds to row j. Use summation notation to write an expression for the following:

   (a) The sum of all matrix elements;
   (b) The sum of the diagonal elements;
   (c) The sum of the elements in the i^{th} row.

2 Exponents and Logarithms

1. Solve the following equations for x:
   (a) $2e^{6x} = 24$;
   (b) $e^{x^2} = 1$;
   (c) $2^x = e^7$;
   (d) $\ln x^2 = 10$;
   (e) $\ln x^{1/2} = \ln 20$.
2. How quickly will \$1000 grow to \$1200 if the interest rate is 5 percent compounded continuously?
3. Derive a formula for the amount of time that it takes for money to triple in a bank account that pays interest at rate r compounded continuously.

3 Calculus

1. Find the derivative with respect to x of the following functions:
   (a) $-9x^4$;
   (b) $\frac{1}{2}\sqrt{x}$;
   (c) $3x^2 - 5x + 2x^{\frac{1}{2}} - 9$;
   (d) $\frac{x}{x^2+1}$;
   (e) $(x+2)^2(4x^2+1)^3$;
   (f) e^{3x};
   (g) $\ln x^2$.
2. Compute the second derivatives of the functions in problem 1.
3. Consider the function $f(x) = x^2 + 2$. Find the equation of the tangent line to the graph of $f(x)$ at $x = 3$.
4. Use the first order condition to find the minimum of the function $f(x) = (x-4)^2 + x$. Use the second order condition to verify that this is indeed a minimum.
5. Consider the function $f(x) = 2x^3 - 45x^2 + 300x + 500$. Find the maximum and minimum of $f(x)$ in the interval $[0, 20]$.
6. Compute all the first and second partial derivatives (including cross-partial derivatives) of the following functions:
   (a) $4x^2y - 3xy^3 + 6x$;
   (b) xy^2;

(c) e^{2x+3y}.

7. Use a Taylor polynomial of order 3 to approximate e^x at the point $x = 0$.

4 Probability

1. Consider a large box filled with 75 balls of equal size and weight. 20 of the balls are blue, 35 are red, 12 are black, and the remaining balls are yellow.

   (a) If you are sampling with replacement, what is the probability that the first two balls you select are both blue. What is the probability that they are both yellow?

   (b) Repeat the previous question in the case in which you are sampling without replacement.

   (c) If you are sampling without replacement, what is the probability that the first two balls you select are the same color?

2. Suppose that the scores of an IQ test for all residents of Champaign are uniformly distributed with a minimum of 50 and a maximum of 200.

   (a) If you select someone at random from the population, what is the probability that their IQ score is less than 95?

   (b) What is the probability of selecting someone with an IQ score somewhere between 170 and 190?